Validation of a 3D Particle Method in the context of Vortex Dynamics and Interaction studies

A. Giovannini[1], **D. Margerit, P. Brancher**
Université Paul Sabatier/Institut de Mécanique des fluides de Toulouse
Allée du professeur Camille Soula, 31400 Toulouse, France

1 Introduction

In the context of Vortex Dynamics, and more specifically of complex interactions between vortex tubes, we present the validation tests for a code based on a Vortex Particle formulation and its detailed comparison with a pseudo-spectral code [1]. The idea of using the vorticity field has been mentioned by P.G. Saffman[2]: *'the vorticity satisfies conservation principles known as Helmholtz laws which allow the vorticity to be followed'*. This approach has three advantages: i) the vorticity is on a compact support which remains compact on time; so we save and concentrate computational time, ii) we avoid the maximum of discretization problems associated with the non-linear term by working in a Lagrangian frame, iii) the algorithm supports very large time-stepping. For the test cases we have chosen the classical Crow geometry with different resolutions (64, 128 and 256) and a vortical Reynolds number of 1500.

2 Main features of the VIC code

The mathematical model[3] for the velocity-vorticity formulation is composed of the Vorticity Transport Equation (VTE) written in conservative form and the coupling equation between vorticity $\boldsymbol{\omega}$ and velocity $\mathbf{V}$. This last equation is the Poisson equation $\Delta\mathbf{B} = -\boldsymbol{\omega}$ for the Potential Vector $\mathbf{B}$ (PV) such that $\mathbf{V} = \nabla \times \mathbf{B}$.

The numerical model (see also Ref.[3] for more details) of the VIC (Vortex In Cell) code has the following steps:

- initially particles are placed only in the vortical zones, and the vorticity field is described by a linear combination of Dirac delta-functions,
- each particle is interpolated on a grid with a very accurate (4^{th} order) kernel and this avoid any dissipative behaviour,
- the potential vector $\mathbf{B}$ is evaluated on the grid along with the velocity and the stretching-tilting term is deduced from finite (4^{th} order) differences,
- these last quantities are interpolated to the particles,
- the particles are pushed with their own velocity and their strength are updated with the stretching-tilting term,
- the diffusion step is finally carried out with a PSE (Particle Strength Exchange) scheme.

3 Validation for counter-rotating tubes

The Crow geometry [4] has been chosen for the richness of the mechanisms occurring during the full process of reconnection of the vortex tubes. We put two counter-rotating tubes in a tri-periodic box and destabilize them symmetrically along the Ox axis in the long wave-length regime of Crow[4]. We have chosen to focus on two aspects:

a) the amplification rate S in the linear regime:

We either process for the maximum slope of the vortex lines, namely $S = \sqrt{\omega_y^2 + \omega_z^2}/\omega_x$, or we evaluate the energy relative to each component of the velocity and of the vorticity. In the inviscid and slenderness theory its theoretical value is[4] $S = 6.413$ and agrees quite well with the results of Table 1.

Resolution	S_{theory}	S_{Ex}	S_{Z_y}	S_{Z_z}	Slope
64	6.413	6.53	6.22	7.60	10.09
128	6.413	7.092	5.72	6.95	8.00
256	6.413	6.19	6.32	6.82	6.601

TAB. 1: Amplification rate for the three resolutions.

[1] giova@cict.fr

b) a detailed comparison between VIC at three resolutions and a pseudo-spectral code[1] at 128^3 resolution:

For the comparisons we use local quantities (extrema of velocity and vorticity), surface-integrals of vorticity (circulations in the half-planes normal and parallel to the tube axis and this characterizes the reconnection process), and volume-integrals for the kinetic energy and the enstrophy (example in Fig.1). The analysis completes the usual snapshot visualization with the kinematic $\lambda 2$ criteria [5] and its methodology is detailed in Ref. [1]. The reconnection stage, the bridging and the threading of the tubes until the formation of the rings and the later viscous dissipation are well described and compared.

The main conclusions of the validation are:

- VIC-64 numerical solution suffers from a lack of resolution leading to differences in the time evolution of the diagnostic variables,
- VIC-128 and SPEC-128 compare very well around the reconnection and it gives a very strong criteria despite that the initial fields are different,
- VIC-256 seems to give a quite converged solution and this allows us to plan higher Reynolds number simulations,
- finally the CPU times are of the same order mainly because VIC uses very large time-steps which are independent of the mesh resolution.

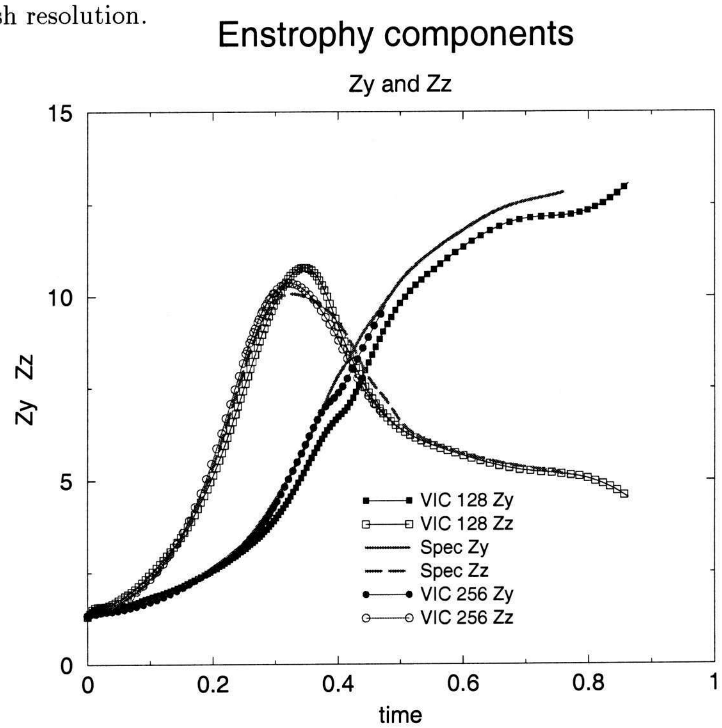

FIG. 1: Time evolution of the Enstrophy components normal to the vortex tubes.

4 Conclusions

All these results contribute to a better knowledge of the stability, accuracy and dissipative behaviour of 3D vortex particle methods. It reinforces the ability of the VIC code to mimic a very complex event in terms of vortex interactions and topology.

Références

[1] J. Marshall, P. Brancher, and A. Giovannini, Interaction of inequal anti-parallel vortex tubes. *J. Fluid Mech.* 446(2001) 229–252.

[2] P. Saffman, *Vortex dynamics.* Cambridge University Press(1992).

[3] G.-H. Cottet and P. Koumoutsakos, *Vortex methods: theory and practice.* Cambridge University Press(2000).

[4] S. Crow, Stability theory for a pair of trailing vortices. *AIAA J.* 8(1970) 2172–2179.

[5] W. Jeong and F. Hussain, On the identification of a vortex. *J. Fluid Mech.* 285(1995) 69–94.